

Final Exam , MTH 221, Spring 2015

Ayman Badawi

QUESTION 1. (10 points). Find the solution set for the the following system of linear equations

$$\begin{aligned} x_1 + 2x_2 &= 1 - x_3 \\ x_2 &= 2 - x_3 \\ x_1 + 3x_2 &= 3 - 2x_3 \end{aligned}$$

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Put the system in the standard form and solve it.
Typical normal question . (so use augmented matrix method)

QUESTION 2.

(4 points). Given $S = span\{(1,0,0,1),(1,1,0,1),(1,1,1,1)\}$. Use Gram-Schmidt Algorithm to find an orthogonal basis for S.

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Typical normal question (just use my class notes). No ideas/ only straight forward calculation

QUESTION 3. (10 points) Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}.$$

(i) Find the inverse matrix of A if it exists.

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Again:)) Nothing new! straight forward calculations.
Just do [A | I_3] cook it until you get [I_3 | A^{-1}]

(ii) Find the inverse matrix of A^T if it exists.

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Note No calculation needed here.You already calculated A^{-1} in (i).
So (A^T)^{-1} = (A^{-1})T

(iii) Find the third column of the inverse matrix of AA^T if it exists.

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Here a new idea I did not mention NOTE (AXB)^-1 = B^-1XA^-1. So (AXA^T)^-1 =
(A^-1)T X A^-1 Now you already calculated both inverses in (i) and (ii).
So third column = (A^-1)T X third column of A^-1

QUESTION 4. (6 points). Let $T : R^3 \longrightarrow R^3$ such that $T(a,b,c) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Then T is a linear transformation (DO NOT SHOW THAT).

(i) Find dim(Range) and write the Range as a span of a basis.

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The given matrix , say M, is the standard matrix representation of T.
So Range = Col(M)/ see class notes

(ii) Does the point $Badawi = (4,5,0)$ belong to the Range of T? If yes, find a point, say $Ayman = (a,b,c)$, such that $T(Ayman) = Badawi$

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To find the point Ayman = (a, b, c)
Solve the system of linear equations MX = (4, 5, 0)^T (M is the given standard matrix rept). If there is a solution, then (4, 5, 0) does belong to the range. If no solution, then (4, 5, 0) does not belong to the range

QUESTION 5. (4 points). Imagine that K is a subspace and $\{k_1, k_2\}$ is a basis for K. Is $\{k_1, k_1 + k_2\}$ a basis for K? Convince me (briefly) that your answer is acceptable.

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We only need show K_1, K_1 + K_2 independent Set a_1K_1 + a_2(K_1 + K_2) = O . Show a_1 = a_2 = 0 (normal zero).
So (a_1 + a_2)K_1 + a_2K_2 = O
(additive identity O). Since K_1, K_2 independent, a_1 + a_2 = 0 and a_2 = 0. Hence a_1 = 0 as well. Done

QUESTION 6. (9 points) For each of the below, if the subset S is a subspace, then rewrite it as a span of some basis, and tell me its dimension. If not a subspace, then give a counter-example.

(i) $S = \{(a, b) \in R^2 \mid (a, b) \text{ is orthogonal to } (2, -1)\}$

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Note S = {(a, b) | 2a - b = 0, a, b in R}.
Hence S = {(a, 2a) | a in R} = span {(1, 2)}.

(ii) $S = \left\{(a, b) \in R^2 \mid a^2 + b^2 \leq 1\right\}$

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Note it cannot be written as span. But here note scalor -maultiplation fails.
Now (1, 0) in S but 2(1, 0) = (2, 0) not in S since a^2 + b^2 = 4 + 0 not less or equal to one

(iii) $S = \{f(x) \in P_3 \mid f(0) = 3\}$

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Note that S = {a_2x^2 + a_1x + 3} not equal span{ } . Another solution x + 3, x^2 + 3 in S but if we add them
, then we get x^2 + x + 6 and if we substitute 0 for x, we get 6 not 3

QUESTION 7. (11 points).

$$A = \begin{bmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \\ p & q & r & s & t \\ u & v & w & x & y \end{bmatrix}$$

and suppose that $\det(A) = \pi$.

(i) Find $\det(A^{-1})$, $det(2A^T)$,

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TRIVIAL QUESTION JUST MY GIFT to all /I mean ALL

(ii) Find the determinant of $B = \begin{bmatrix} 1 & b & c & d & e \\ 0 & 1 & h & i & j \\ 0 & 0 & 1 & n & o \\ 0 & 0 & 0 & 1 & t \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$,

For the matrix B above, what are the eigenvalues of B? Assume that B is is diagnolizable (Do not show that), for each eigenvalue a of B find $.dim(E_a)$

(iii) Find the determinant of $\begin{bmatrix} 1 & b & c & d & e \\ 0 & 1 & h & i & j \\ 0 & 0 & 1 & n & o \\ 0 & 0 & 0 & 1 & t \\ 1 & b & c & d & 3+e \end{bmatrix}$.

QUESTION 8. (16 points) Determine whether each statement is true or false and give a brief justification for your answer (should not exceed two CLEAR MEANINGFUL lines)

(i) If A is a 3×3 invertible matrix, then A is diagnolizable.

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False/

(ii) It is impossible to construct a linear transformation $T : R^2 \to R^4$ such that $dim(Range(T)) = 3$.

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True, since dim(Ker) + dim(range) = dim (domain)

(iii) If A is a 4×4 matrix and 1 is an eigenvalue of A, then there is a nonzero 4×10 matrix B, such that $AB = B$.

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True We KNOW AQ^T = Q^T for some point Q in R^4.. Thus Let B = [10 column , each column is Q^T] :))))

(iv) If $T : R^3 \to R$ is a linear transformation and $T(1, 4, 7) = \pi$, then $dim(Ker(T)) = 2$.

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True/

(v) If A is a 4×5 matrix and $Rank(A) = 4$, then the system $AX = \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}$ has infinitely many solutions

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True. Since dim(col(A)) = 4, Col(A) = R^4. Hence every point in R^4 is a linear combination of columns of A.

(vi) If A is a 3×3 matrix such that $det(A) = 0$, then the system $AX = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ has infinitely many solutions

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False/ Maybe infinte or no solution

(vii) If A is a 4×4 matrix and the system $AX = \begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix}$ is inconsistent (i.e., it has no solution), then the system $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ has infinitely many solutions.

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True. Because |A| = 0 and every homogeneous system is consistent and thus must have infinite solution

(viii) If A is a 4×4 matrix and $C_A(x) = x^2(x-3)^2$, then the system $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ has infinitely many solutions.

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True, Since 0 is an eigenvalue of A, |A| = 0. Thus the homogenous system has infinitely many solutions

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Q1 Let $M = \begin{bmatrix} -1 & -2 & 1 & 3 \\ 1 & 0 & 1 & -1 \\ 2 & 1 & 2 & 1 \end{bmatrix}$. Then the COMPLETE reduced form of of M is

A) $\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & -2 & 2 & 2 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

E) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$\boxed{Q2}$ Let M be a $n \times n$- matrix such that $\det(M) \neq 0$. Which of the following statements is **always true**

A) M is diagonalizable

B) M has n distinct eigenvalues

C) 0 is an eigenvalue of M

D) It is possible that 0 is an eigenvalue of A^T

E) All eigenvalues of M are nonzero

Q3 If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is such that $\det A = 4$, then the determinant

$$\begin{vmatrix} a-2d & b-2e & c-2f \\ \frac{1}{2}g & \frac{1}{2}h & \frac{1}{2}i \\ 2d & 2e & 2f \end{vmatrix}$$ is equal to

A) 8

B) 4

C) 2

D) -8

~~E)~~ -4

$\boxed{Q4}$ Consider the following subsets of $\mathcal{P}_3$:

$$R = \{f(x) \in \mathcal{P}_3 : \ f'(2) = 0\}, \ S = \{f(x) \in \mathcal{P}_3 : \ f(1) \geq 0\}$$

$$\text{and } T = \{f(x) \in \mathcal{P}_3 : \ f(x) + f'(x) = 0\}.$$

Which of these subsets is a subspace of $\mathcal{P}_3$?

A) R, S, and T

~~B)~~ R and T only

C) T only

D) S only

E) R only

$\boxed{Q5}$ Recall that a square matrix A is said to be symmetric if $A^t = A$. If A is a square matrix, then

A) AA^t and $A - A^t$ are symmetric

B) $A + A^t$ and $A - A^t$ are symmetric

~~C)~~ AA^t and $A + A^t$ are symmetric

D) AA^t, $A + A^t$ and $A - A^t$ are symmetric

E) AA^t, $A + A^t$ and $A - A^t$ are not symmetric

$\boxed{Q6}$ Which of the following sets is a basis for $\mathcal{P}_3$

A) $\{1 + x + x^2, 1 + 2x + 2x^2, -2 - 3x - 3x^2\}$

B) $\{1 + x + x^2, x + x^2, 2\}$

C) $\{x + x^2, x + 1, -x^2 + 1\}$

~~D)~~ $\{1 + x + x^2, x + x^2, x^2\}$

E) $\{1, 1 + x + x^2\}$

Q7 If the point $(1, a, b) \in span\{(1, 1, 0), (2, 1, 1), (2, 3, -1)\}$. Then

A) $a = 0$ and $b = 2$

B) $a = 1$ and $b = 1$

C) $a = 1$ and $b = -1$

D) $a = 2$ and $b = 1$

~~E)~~ $a = 2$ and $b = -1$

$\boxed{Q8}$ Let $T : \mathbb{R}^4 \to \mathbb{R}^3$,

$$T(a,b,c,d) = \begin{bmatrix} -3 & 1 & 3 & -2 \\ 1 & 1 & -3 & 4 \\ 1 & 3 & -5 & 8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

.

A) dim $Range(T) = 2$

B) dim $Ker(T) = 0$

C) $(-3, 3, 5)$ is not in $Range(T)$

D) $Range(T) = \mathbb{R}^3$

E) $Ker(T) = span\{(0, 1, 1, 0)\}$

Q9 Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation such that $Kert(T) = \{(0, 0, ..., 0)\}$ and $Range(T) = R^m$. Let M be the standard matrix representation of T. then

A) $n < m$ and $dim(Row(M)) = n$

B) $n > m$ and $dim(Col(M)) = m$

C) It is possible that $det(M) = 0$.

D) $n = m$

E) It is impossible that $M = M^T$

Q10 Let $T : R^2 \to P_2$ be a linear transformation such that $T(1,1) = x$ and $T(-1,1) = 2$. Then $T(0,4) =$

~~A)~~ $2x + 4$

B) $x + 4$

C) $2x + 2$

D) 4

E) $4x + 8$

$\boxed{Q11}$ The following system of linear equations:

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 0 \end{bmatrix}$$

A) has a unique solution

B) has infinitely many solutions

C) has $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ as a solution

~~D)~~ has no solution

E) has $\begin{bmatrix} -5 \\ 0 \\ 0 \end{bmatrix}$ as a solution

$\boxed{Q12}$ Let $T : R^2 \to \mathbb{P}_2$ be a linear transformation such that $T(a,b) = (a+3b)x + (2a+6b)$. Then the fake standard matrix representation of T is

A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

~~B)~~ $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

C) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

D) $\begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$

E) $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

$\boxed{Q13}$ Let T as above then:

A) $Ker(T) = \{(0,0)\}, Range(T) = span\{x\}$

B) $Ker(T) = \{(0,0)\}, Range(T) = Span\{x+2\}$

C) $Ker(T) = span\{(1,-3)\}, Range(T) = Span\{x+2\}$

D) $Ker(T) = span\{(-3,1)\}, Range(T) = Span\{x\}$

~~E)~~ $Ker(T) = span\{(-6,2)\}, Range(T) = Span\{x+2\}$

Q14 Let $M = \begin{bmatrix} a^2 & a^3 \\ 1 & a^4 \end{bmatrix}$. Which of the following statements is **always true**

~~A)~~ When M is invertible, $M^{-1} = \begin{bmatrix} \frac{a}{a^3-1} & \frac{-1}{a^3-1} \\ \frac{-1}{a^3(a^3-1)} & \frac{1}{a(a^3-1)} \end{bmatrix}$

B) det $M = 0$ only if $a = 1$

C) M is invertible only if $a \neq 0$

D) When M is invertible, $M^{-1} = \begin{bmatrix} \frac{1}{a(a^3-1)} & \frac{-1}{a^3-1} \\ \frac{-1}{a^3(a^3-1)} & \frac{a}{a^3-1} \end{bmatrix}$

E) M is row equivalent to I_2

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